# A ONE DIMENSIONAL (1D) NON –STATIONARY MODEL FOR CATHODE SPOT OF THE METAL VAPOR ARC


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## 1. Introduction

One of the principal property of the cathode spot is changes in a nanosecond times [**1**], that seems a very similar to a single explosive centre [**2**] with the same electrical current. Developed for the single emission centre 1D model may be used for explanation of the spot dynamics and after the proper explosion stage – its ignition.

## 2. Equations

### 2.1 Non-stationary heat conduction with spherical symmetry and displacement of the melted surface

We assume spherical symmetry with $R_c$ – radius of emitting area and $R_m$ – radius of melted area. With some simplification we treat the problem one-dimensionally.
Non–stationary heat conduction are described by

$$\rho c \frac{\partial T}{\partial t} = \frac{\lambda}{R^2}\frac{\partial}{\partial R}\left(R^2 \frac{\partial T}{\partial R}\right) - \frac{c_e I_a}{2\pi R^2 e}\frac{\partial T}{\partial R} + \left(\frac{I_a}{2\pi R^2}\right)^2 \kappa_0 T, \tag{1}$$

with the initial and boundary conditions for heat transition through the surface and into the metal bulk are

$$\begin{aligned} &T\big|_{t=0} = T_0, \quad T\big|_{R\to\infty} = T_0, \quad T_c = T(R_c, t), \\ &-\lambda_m \nabla T\big|_{R=R_c} = Q_s(T_c, U_c). \end{aligned} \tag{2}$$

The displacement of the emitting surface for $t>t_0$ reads

$$\begin{aligned} &\frac{\partial R_c}{\partial t} = V_p + V_e, \quad R_c\big|_{t=t_0} = R_{c0}(I_a), \\ &V_p = \left(P_c(T_c, U_c)/2\rho\right)^{1/2}, \quad V_e = M_a I_{v0}\cdot(1-\beta)/\rho, \end{aligned} \tag{3}$$

where $\beta(T_c,U_c)=1 - I_v/I_{v0}$ describes the returning part of the evaporated flow, $t_0$ is ignition time.
The radius of the solid-liquid transition region is described by

$$\begin{aligned} &\lambda_s \nabla T\big|_s - \lambda_m \nabla T\big|_m = \rho E_m V_m, \qquad V_m = \frac{\partial R_m}{\partial t}, \\ &R_m\big|_{t=t_0} = R_{m0}(I_a), \quad T(R_m, t) = T_m. \end{aligned} \tag{4}$$

The equation for the electric current yields a parametric dependence to find $U_c(t)$:


Proceedings of 24th International Conference on Phenomena in Ionized Gases (**ICPIG**),
Warsaw, Poland, July 1999, v.2, pp.177-178.

$$I_a(t) = 2\pi R_c^2 J_c(T_c, U_c) = \text{const}\,. \quad (5)$$

We use the following quantities: $c_e=\pi^2k^2T/2\mu$ – specific electron heat capacity, $\mu=E_f[1-(\pi^2/12)(kT/E_f)^2]$ – chemical potential, $E_f$ – Fermi energy, $\kappa_0$ – temperature coefficient of the resistivity, $E_m$ – specific melting heat; $T_m$ – melting temperature; $T_0$ is the temperature of the cathode bulk metal; subscripts $m$ and $s$ correspond to liquid (melted) and solid state, respectively; $V_p$ and $V_e$ are the velocity components of the liquid surface associated with the hydrodynamic movement due to the plasma pressure and the erosion at evaporation; $I_{v0}$ and $I_v$ are the evaporation flow at the surface and at the distant border of the Knudsen layer, $M_a$ is the atomic weight. As material parameters we take density $\rho$=8.9 g/cm$^3$, specific heat capacity c=0.384 J/gK, and heat conductivity $\lambda$=3.4 W/Kcm. In principle these parameters depend on the temperature, but as a first approximation this dependence is neglected.

## 2.2 Boundary conditions on the plasma side

Presented above system of equations is practically the same as for the classic emission–erosion model of the single explosive emission centre [2]. The differences in results of numerical experiments caused by the used in our calculations the self–consistent boundary conditions at the stage followed to proper explosion. The physical model for the near cathode plasma layer and calculated for copper cathode boundary conditions for the equations set (1)–(5) was presented in the [3]: energy flow to the surface $Q_s(T_c, U_c)$, pressure in the near–cathode plasma ball $P_c(T_c, U_c)$, returning part of evaporated particles $\beta(T_c, U_c)$, and current density $J_c(T_c, U_c)$.

## 3. Results

Figs.1–3 shows the results for the 1D model. The spot currents were chosen to $I_a$=10A, 20A, 30A, 40A, and 80 A for a Cu-cathode at $T_0$=300 K.

It has been assumed that the spot formation starts by Joule heating of a small area with initial radius $R$~0.1µm by the current increasing with a rate of up to $10^9$ A/s. At ignition the current density is >$10^8$ A/cm$^2$. It decreases by about one order of magnitude when reaching the surface heating stage characteristic for the fully developed spot (fig. 2*a*). After $t_0$~1÷3 ns the required current can be sustained by a finite cathode fall (Fig.2 *b*).

This time marks the transition from the explosive erosion due to Joule heating (volume energy source) to the proper cathode spot characterised mainly by ion bombardment heating (surface energy source).

This assumption about the transition from Joule heating to the surface energy source may be supported by the behaviour of the effective part of the total energy $U_c \cdot I_a$, dissipated in the cathode bulk, shown in Fig.1 *a*. For times <1ns Joule heating is prevailing. In this stage about 20 % of the arc power is dissipated in the cathode bulk. To reach the experimental value of ≅0.3, times >10 ns must elapse. At this stage the contribution of Joule heating can be neglected.

At the same time the temperature decreases from values > 6000 K down to values <5000 K

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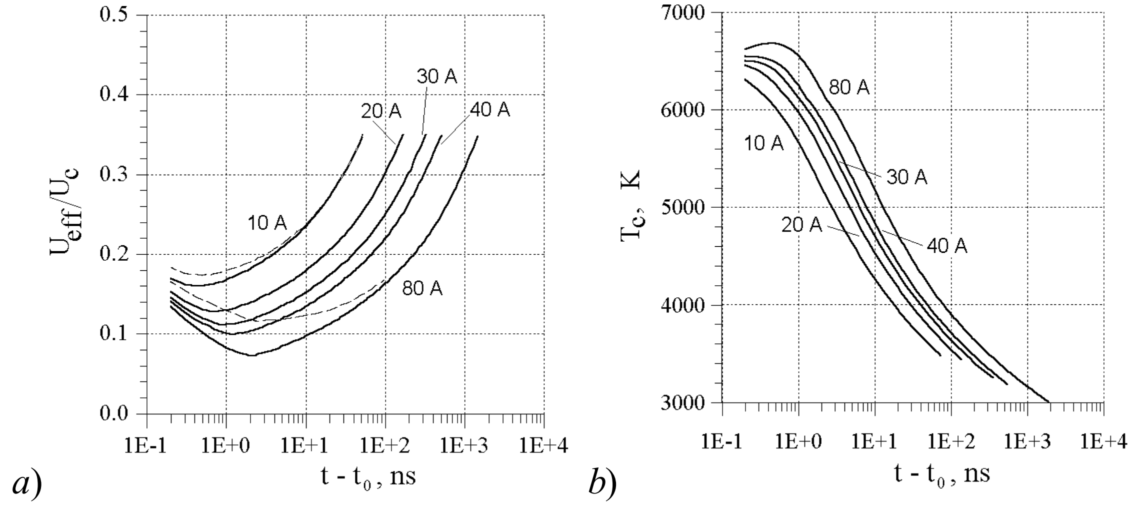


*Fig. 1. The time dependence of the fraction of the energy $I_aU_c$ spent on cathode processes (a) and the surface temperature $T_c$ (b) for the fixed values of the spot current $I_a$. The dashed lines depict the case when Joule heating is taken into account.*

(Fig 1 *b*).

If we define the characteristic spot stage by values of the cathode fall <20 V, Fig. 1–2 yields another interesting result: The spot has a finite lifetime depending on the current. It amounts to 10 ns for 10 A, and to 500 ns for 80 A. As a consequence, for longer arc burning times there must be a sequence of spot reignitions.

In [**4**, **5**, **6**] experimental data on the chaotic motion of cathode spots are reported that little depend on the current. According to Fig. 3 *a* for $t$>10 ns the radius $R_c$ can be approximated by $\lg(R_c) \cong p\cdot\lg(t) + q'(I_a)$, and $p\cong 1/2$. Thus, $\lg(R_c^2/t) \cong q(I_a) = 2\cdot q'(I_a)$. Changes of $q$ with $I_a$ are small for $t$>10 ns. For currents $I_a\in[10, 100]$ A the values of the diffusion constant of random walk over the surface $\langle X^2\rangle$ on the time τ are then $D = \langle X^2\rangle/\tau \approx R_c^2/t \cong \text{const} \approx 10^{-3}$ $m^2$/s, in accordance with the experiments.

Fig. 3 *b* shows the averaged displacement velocity $V_s=(R_c - R_{c0})/(t-t_0)$ of the liquid surface, amounting to 20 – 200 m/s. Interestingly, it depends on the current only during the ignition phase.

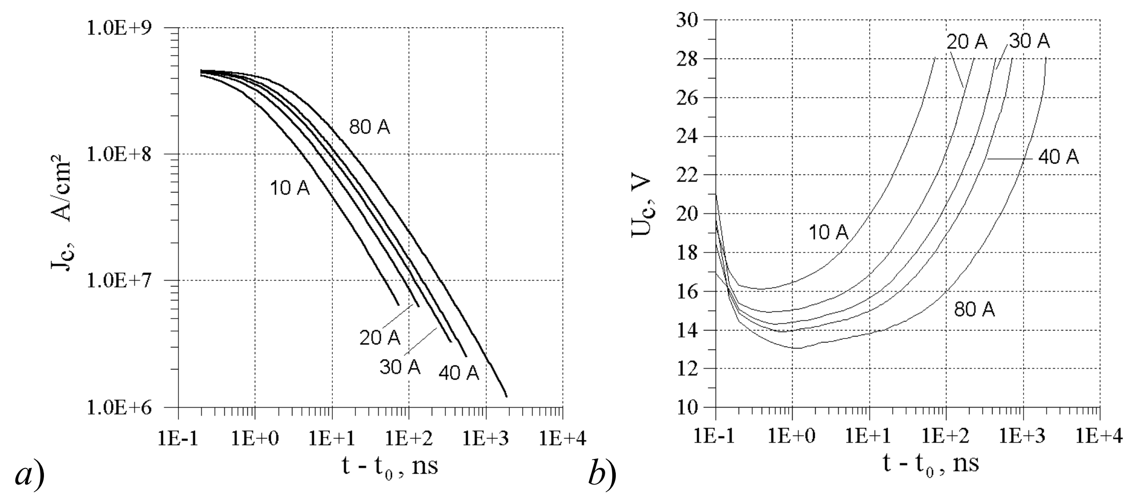


*Fig. 2. The time dependence of the current density $J_c$ (a) and cathode fall $U_c$ (b) for fixed values of the spot current $I_a$.*

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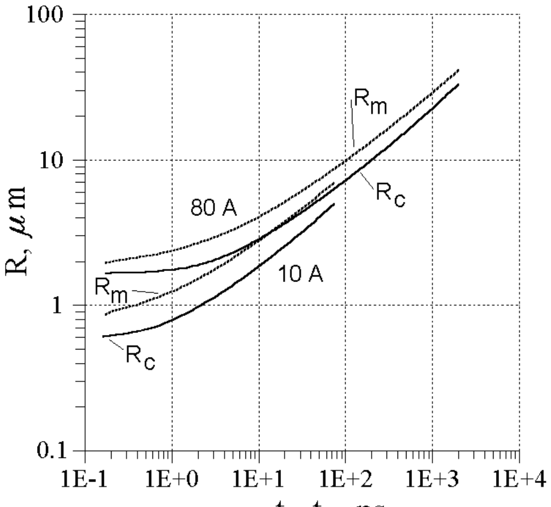
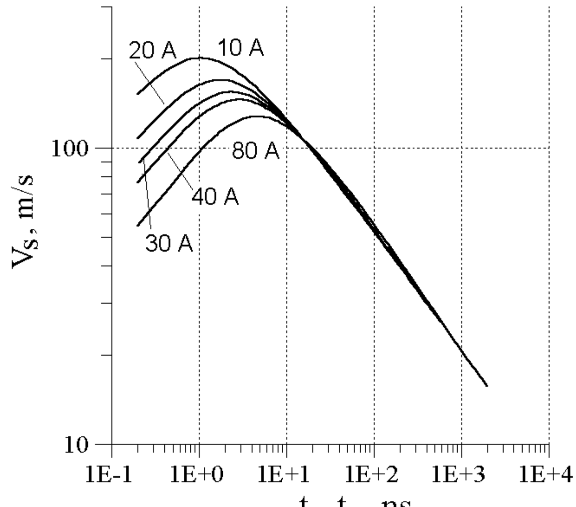



*a*)




*b*)

*Fig. 3. The time dependence of the emitting surface radius $R_c$ and the radius $R_m$ of the melted zone* (*a*) *and the surface velocity* (*b*).

## 4. Summary

1D modelling of the heat conduction and surface depression within the cathode spot yields information on the time scale of the spot processes and of the spot size. Typical times amount to 1–100 ns, typical radii to 1–10 μm. Also, minimum spot currents can be calculated (1–10 A) as well as maximum spot lifetimes (10 ns – 500 ns). Finally, from the temporal evolution of the spot radius the diffusion parameter of random spot movement is obtained to about $10^{-3}$ $m^2/s$, in agreement with the experiment.

## 5. Acknowledgments


The author wish to thanks to Professor B. Jüttner for a generous support.
This work was supported by the German Ministry of Sciences BMBF, grant 13N6616.


## 6. References


[1] B. Jüttner : General Invited Lecture, 23th ICPIG, Toulouse, In: J. Phys. IV France (1997) C4-31.
[2] G.A. Mesyats and D.I. Proskurovsky : Pulsed Electrical Discharge in Vacuum. Heidelberg: Springer, 1989.
[3] Yu. Vasenin : Proc. of 18th ISDEIV, Eindhoven, The Netherlands (1998) 341.
[4] B. Jüttner : Beitr. Plasmaphys., 19 (1979) 25.
[5] B. Jüttner : J. Phys. D: Appl. Phys., 14 (1979) 1265.
[6] V.F. Puchkarev, A.M. Murzakayev : J. Phys. D: Appl. Phys., 23 (1990) 26.



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Warsaw, Poland, July 1999, v.2, pp.177-178.